SPACETIME DISCONTINUOUS GALERKIN METHOD FOR SYSTEMS OF NONLINEAR CONSERVATION LAWS[1]

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We present a spacetime discontinuous Galerkin (SDG) finite element method for systems of nonlinear conservation laws. The method extends concepts introduced by Lowrie et al. [1]. We use a weighted residual statement, defined in terms of spacetime BV functions, to enforce the integral form of the conservation law on each element. We define conservation in terms of the physical Godunov fluxes obtained from the local Riemann problem, and show that the use of the Godunov fluxes in the continuum SDG formulation delivers a priori satisfaction of the entropy condition. The Rankine-Hugoniot conditions arise naturally as the spacetime jump conditions that couple the solutions in adjacent elements.

The SDG method is compatible with either structured or unstructured spacetime meshes. If the spacetime grid conforms to a special causality cone constraint, then a direct element-by-element or patch-by-patch solution procedure with $\mathcal{O}(N)$ complexity is possible. However, since the cone constraint is nonlinear and solution-dependent, we use an advancing-front solution technique in which mesh generation is interleaved with a patch-wise finite element solution procedure. The patch-wise meshing algorithm for nonlinear cone constraints is an extension of the tent-pitcher algorithm [2].

The new method delivers solutions that are free of Gibbs ringing without straying from the standard Bubnov-Galerkin framework; no stabilization is required. However, some under/overshoot might still occur in the immediate vicinity of shocks. We discuss limiters that can control under/overshoot on different classes of spacetime meshes. In addition, we consider an expanded family of discontinuous Galerkin basis functions that might eliminate the need for limiting altogether. We demonstrate the performance of the SDG finite element method for applications involving systems of nonlinear conservation laws in two dimensions.

References

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